

Topics : Center of Mass, Work, Power and Energy, Rigid Body Dynamics

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.4

(3 marks, 3 min.)

M.M., Min.

[12, 12]

Subjective Questions ('-1' negative marking) Q.5 to Q.6

(4 marks, 5 min.)

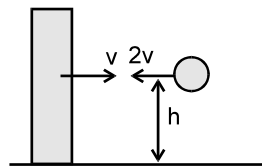
[8, 10]

Assertion and Reason (no negative marking) Q.7 to Q.9

(3 marks, 3 min.)

[9, 9]

1. A ball collides elastically with a massive wall moving towards it with a velocity of v as shown. The collision occurs at a height of h above ground level and the velocity of the ball just before collision is $2v$ in horizontal direction. The distance between the foot of the wall and the point on the ground where the ball lands, at the instant the ball lands, will be :



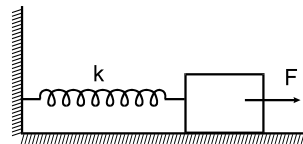
(A) $v\sqrt{\frac{2h}{g}}$

(B) $2v\sqrt{\frac{2h}{g}}$

(C) $3v\sqrt{\frac{2h}{g}}$

(D) $4v\sqrt{\frac{2h}{g}}$

2. A block attached to a spring, pulled by a constant horizontal force, is kept on a smooth surface as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is : [Given that spring does not break]



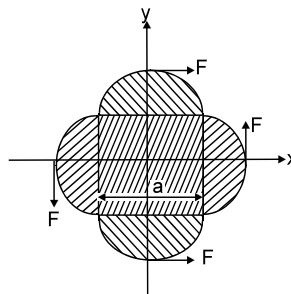
(A) $\frac{F^2}{K}$

(B) $\frac{2F^2}{K}$

(C) ∞

(D) $\frac{F^2}{2K}$

3. A planar object made up of a uniform square plate and four semicircular discs of the same thickness and material is being acted upon by four forces of equal magnitude as shown in figure. The coordinates of point of application of forces is given by



(A) $(0, a)$

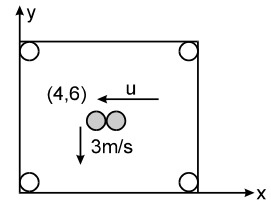
(B) $(0, -a)$

(C) $(a, 0)$

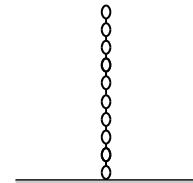
(D) $(-a, 0)$

4. The angular velocity of a rigid body about any point of that body is same:
 (A) only in magnitude
 (B) only in direction
 (C) both in magnitude and direction necessarily
 (D) both in magnitude and direction about some points, but not about all points.

5. On a smooth carrom board, a coin located at (4, 6) is moving in negative y-direction with speed 3 m/s is being hit at that point by a striker moving along negative x-axis. The line joining centre of the coin and striker just before the collision is parallel to x-axis. After collision the coin goes into the hole located at origin. Mass of the striker and the coin is equal. Considering the collision to be elastic, find the velocity (in vector form) of the striker before the collision and after the collision.

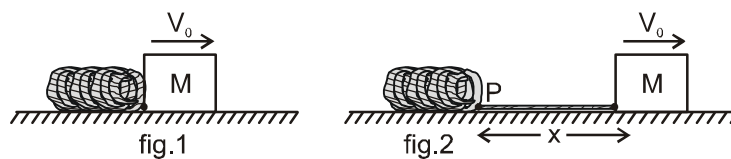


6. A uniform chain of mass m and length ℓ hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the table on the chain when half of its length has fallen on the table. The fallen part does not form heap.



COMPREHENSION

A smooth rope of mass m and length L lies in a heap on a smooth horizontal floor, with one end attached to a block of mass M . The block is given a sudden kick and instantaneously acquires a horizontal velocity of magnitude V_0 as shown in figure 1. As the block moves to right pulling the rope from heap, the rope being smooth, the heap remains at rest. At the instant block is at a distance x from point P as shown in figure-2 (P is a point on the rope which has just started to move at the given instant), choose correct options for next three question.



7. The speed of block of mass M is

(A) $\frac{mV_0}{(M + \frac{m}{L}x)}$ (B) $\frac{MV_0}{(M + \frac{m}{L}x)}$ (C) $\frac{m^2V_0}{M(M + \frac{m}{L}x)}$ (D) $\frac{M^2V_0}{m(M + \frac{m}{L}x)}$

8. The magnitude of acceleration of block of mass M is

(A) $\frac{m^3}{L} \frac{V_0^2}{(M + \frac{m}{L}x)^3}$ (B) $\frac{mM^2}{L} \frac{V_0^2}{(M + \frac{m}{L}x)^3}$ (C) $\frac{m^4}{ML} \frac{V_0^2}{(M + \frac{m}{L}x)^3}$ (D) $\frac{M^2}{L} \frac{V_0^2}{(M + \frac{m}{L}x)^3}$

9. The tension in rope at point P is

(A) $\frac{mM^2}{L} \frac{V_0^2}{(M + \frac{m}{L}x)^2}$ (B) $\frac{m^2M}{L} \frac{V_0^2}{(M + \frac{m}{L}x)^2}$ (C) $\frac{m^3}{L} \frac{V_0^2}{(M + \frac{m}{L}x)^2}$ (D) $\frac{M^3}{L} \frac{V_0^2}{(M + \frac{m}{L}x)^2}$

Answers Key

1. (C) 2. (B) 3. (B) 4. (C)
 5. (2,0) 6. $\frac{3}{2}$ 7. (B) 8. (B)
 9. (A)

Hint & Solutions

1. Solve in the reference frame fixed to the wall.
 Before collision, velocity of ball = $3v$ towards it.
 \therefore After elastic collision of ball = $3v$ away from it

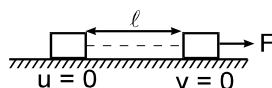
$$\text{Time of flight} = \sqrt{\frac{2h}{g}}$$

\therefore distance between wall and ball

$$= 3v \cdot \sqrt{\frac{2h}{g}}$$

(Here no pseudo force is applied since the wall keeps on moving with constant velocity w.r.t ground, it being very heavy.)

2. **(B)** Applying work energy theorem on block



$$F\ell - \frac{1}{2} k\ell^2 = 0 \quad \therefore \quad \ell = \frac{2F}{k}$$

$$\text{or work done } I = F\ell = \frac{2F^2}{k}$$

3. **(B)** The two forces along y-direction balance each other.

Hence, the resultant force is $2F$ along x-direction

Let the point of application of force be at $(0, y)$.

(By symmetry x-coordinate will be zero).

For rotational equilibrium :

$$F(a) + F(a) + F(a + y) - F(a - y) = 0$$

$$\Rightarrow y = -a \quad \text{Hence (B).}$$

$$F(-a) + F(-a) + F(-a + y) - F(-a - y) = 0$$



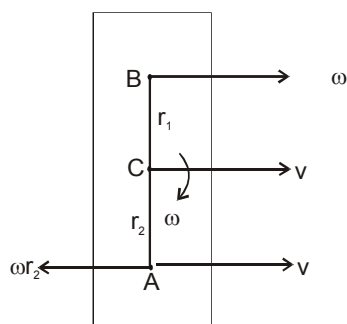
Alternate :

Torque will only be produced by the two forces along y-direction in anti-clockwise direction. To balance this torque we should apply a force 2F in order to produce a torque in the clockwise direction, which is only possible if we apply a force at a point below the x-axis.

Then , $\tau = F(a) + F(a) - 2F \times y = 0$

$\Rightarrow y = a$ Hence (B).

4. Suppose a rod is having angular velocity ω about point C .



Choose two points A and B as shown in the fig. velocity of B w.r.t A = $(v + \omega r_1) - (v - \omega r_2)$

$\Rightarrow V_{BA} = \omega(r_1 + r_2)$

Angular velocity of B w.r.t A = $\frac{V_{BA}}{AB}$

$= \frac{\omega(r_1 + r_2)}{r_1 + r_2} = \omega$ **Ans (C)**

5. The line of impact for duration of collision is parallel to x-axis.

The situation of striker and coin just before the collision is given as

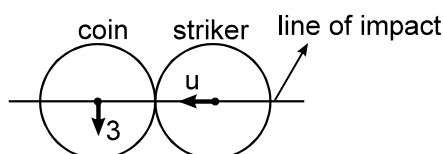
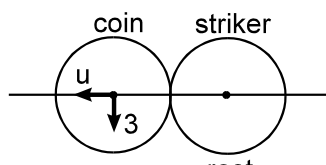
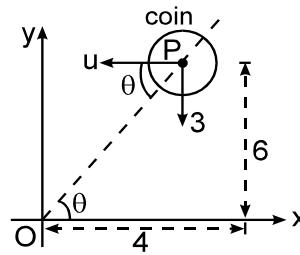


Figure (A) before collision



Because masses of coin and striker are same, their components of velocities along line of impact shall exchange. Hence the striker comes to rest and the x-y component of velocities of coin are u and 3 as shown in figure.



For coin to enter hole, its velocity must be along PO

$$\therefore \tan \theta = \frac{6}{4} = \frac{3}{u}$$

or $u = 2 \text{ m/s}$ **Ans. (2, 0)**

6. 1. Weight of the portion BC of the chain

lying on the table, $W = \frac{mg}{2}$ (downwards) Using

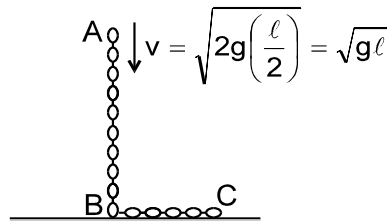
$$v = \sqrt{2gh}$$

2. Thrust force $F_t = v_r \left(\frac{dm}{dt} \right)$

$$v_r = v$$

$$\frac{dm}{dt} = \lambda v$$

$$F_t = \lambda v^2$$



(where, $\lambda = \frac{m}{l}$, is mass per unit length of chain)

$$v^2 = (\sqrt{gl})^2 = gl$$

$$\therefore F_t = \left(\frac{m}{l} \right) (gl) = mg \quad (\text{downwards})$$

\therefore Net force exerted by the chain on the table is

$$F = W + F_t = \frac{mg}{2} + mg = \frac{3}{2}mg$$

So, from Newton's third law the force exerted

(vertically upwards).

7.to 9 The mass of moving material is $M + \frac{m}{L}x$.

From conservation of momentum MV_0

$$= (M + \frac{m}{L}x)V$$

\therefore velocity of moving block and moving rope is

$$V = \frac{0}{(M + \frac{m}{L}x)}$$

8. (B) The acceleration of moving block is

$$a = -v \frac{dv}{dx} = - \frac{MV_0}{(M + \frac{m}{L}x)^2} \times \frac{m}{L} \frac{dx}{dt}$$

$$= - \frac{m}{L} \frac{M^2 V_0^2}{(M + \frac{m}{L}x)^3}$$

9. (A) The tension at point P is what gives momentum to next tiny piece (to left of P) that starts moving. The speed of this piece increases from 0 to V in time dt.

$$\Rightarrow dp = dmV$$

$$\text{or } F = \frac{dP}{dt} = \frac{dm}{dt}V = \frac{\frac{m}{L}dx}{dt}V = \frac{m}{L}V^2$$

$$\therefore F_p = \frac{m}{L} \frac{M^2 V_0^2}{(M + \frac{m}{L}x)^2}$$